NITM Mathematical Bi-monthly

October-November 2024

Department of Mathematics,
National Institute of Technology Meghalaya
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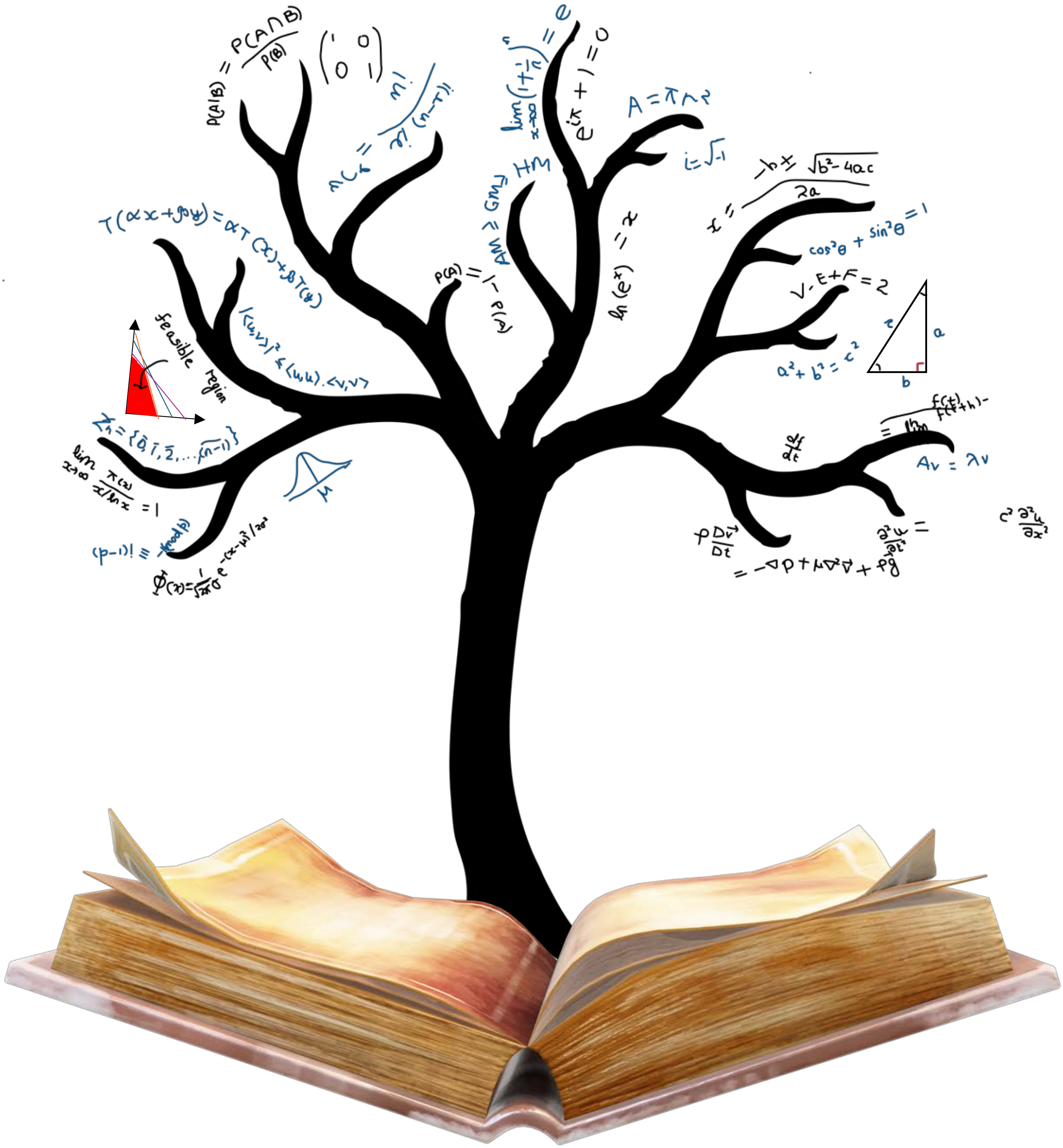
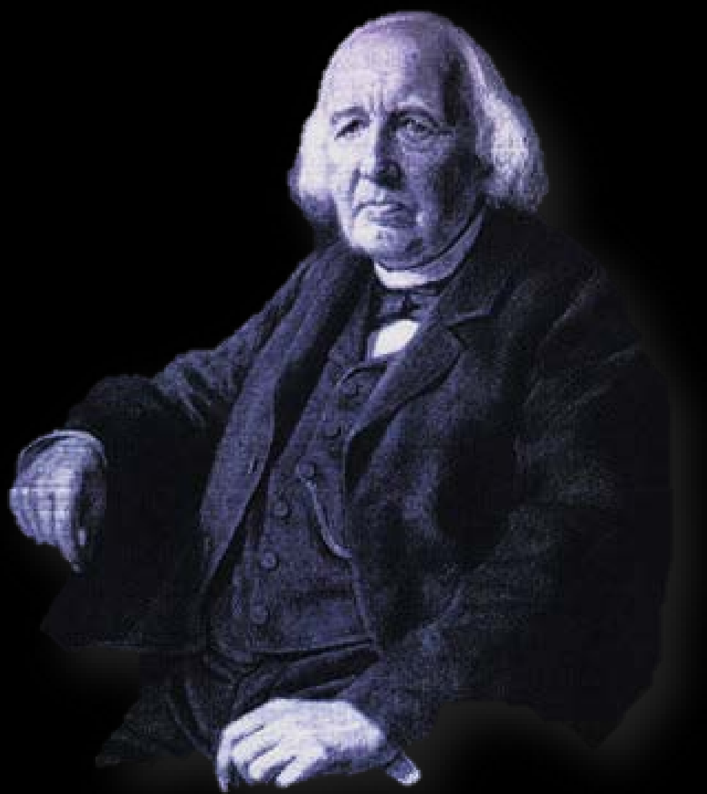


P(A|B) = P(A∩B) / P(B)
e^{iπ} + 1 = 0
A = πr²
i = √-1
AM ≥ GM ≥ HM
V - E + F = 2
cos²θ + sin²θ = 1
a² + b² = c²
Av = λv
feasible region



“A mathematician who is not also something of a poet will never be a complete mathematician.”

— **Karl Weierstrass**

(31 October 1815-19 February 1897)

DIRECTOR'S MESSAGE

Dear Students, Faculty, and Readers,

I am immensely pleased to introduce the 2nd edition of the Department of Mathematics' bimonthly magazine. This magazine represents a significant step forward in creating a platform where the department can showcase our students and faculty members' intellectual curiosity, talent, and dedication.

Mathematics is not just a subject confined to classrooms and textbooks; it is a dynamic and evolving field with the power to shape the world around us. I am proud of the department's commitment to fostering academic excellence and a spirit of innovation.

This magazine is a testament to that pursuit of knowledge. It will serve as a medium for not only disseminating new ideas and research but also for encouraging discussions, collaborations, and creativity within our vibrant mathematical community. I encourage each of you—students and faculty alike—to contribute actively to the growth of this magazine and make it a reflection of our collective brilliance.

As we move forward, let us continue to strive for academic distinction, intellectual curiosity, and a passion for solving the complex problems that mathematics presents. The journey is as important as the destination. I believe that together, we will continue to make strides toward a brighter future for the department and the world of mathematics.

I congratulate the editorial team on their hard work in bringing this publication to life, and I look forward to seeing the magazine evolve in the years to come.

With best wishes,

Prof. Pinakeswar Mahanta

Director, NITM

HoD's Message

As the Head of the Department of Mathematics (MA), it is truly an honor for me to write for the 2nd edition of our departmental magazine. This platform allows our faculty, staff, and students to showcase their achievements, express their opinions, and explore new mathematical concepts. I believe that this magazine will become an essential channel for sharing knowledge, ideas, and research insights that will inspire us all.

The Department of Mathematics, which started functioning in June 2012, currently offers 2-year M.Sc. and PhD programs. M.Sc. students are selected based on their ranking through CCMN and the institution mode, while PhD students are selected through interviews based on their GATE/NET scores. In addition to our core programs, the department also plays a vital role as a supporting pillar for various B. Tech and M. Tech programs within the institute.

The creation of this magazine stems from a collective desire to share our thoughts, accomplishments, and aspirations. Working together as a team to ensure its successful publication brings immense delight. I feel privileged to be part of this process and am filled with joy in nurturing our students, contributing to society, and fostering academic excellence.

My team and I remain dedicated to the holistic development of our students within the institute. I extend my best wishes to all MA family members and sincerely hope that this tradition of the departmental magazine continues for generations to come, fostering happiness, unity, and intellectual growth.

Warm regards,
Dr. Adarsha Kumar Jena
Assistant Professor, HoD, MA

Editor’s Message

The only way to learn mathematics is to do mathematics. — Paul R. Halmos.

This profound statement not only serves as a guiding principle but also emphasizes the importance of active engagement in mathematics. It brings me great joy to inform you that the Department of Mathematics at the National Institute of Technology Meghalaya is introducing its 2nd edition of its own publication the “*NITM Mathematical Bi-monthly.*”

This publication is a collective endeavor by our students and faculty members, designed to ignite a love for mathematics and offer a stage for students to share their insights. Magazines transform the creative potential of our students into tangible contributions, allowing them to identify and showcase their talents through writing. Through this magazine, we aspire to highlight contributions, departmental events, achievements, and the scholarly work of both faculty and students. I encourage all students to participate by submitting interesting mathematical problems, engaging puzzles, stories, and intriguing facts about mathematicians.

I want to express my deepest appreciation to the editorial team—Bankit, Sanchita, Dixita, and Dibyasman—for their tremendous dedication and hard work in making this magazine a reality in such a brief period. Our minds are filled with boundless curiosity, and we are continually striving to explore beyond the known. I wish all our students’ immense success as they delve into the magazine's contents and set out on fresh intellectual journeys. May this initiative inspire us all to deepen our grasp of mathematics with steadfast determination.

Thank you, and best wishes.

Dr. Timir Karmakar
Assistant Professor
Department of Mathematics

Buffon Needle Problem

Finding π is a child's play

Problem statement:

Given a floor with equally spaced parallel lines (a distance 'd' apart), what is the probability that a matchstick/needle of length L will land on a line?

Answer: $\frac{2L}{\pi d}$

History:

⊙ The problem was first posed by French naturalist and mathematician Georges-Louis Leclerc, Comte de Buffon in 1733 and reproduced with solution by himself in 1777.

⊙ In 1901, An Italian mathematician Mario Lazzarini also performed Buffon's needle experiment tossing a needle 3408 times and obtained the well-known estimate 355/113 for π, accurate to six significant figures.

A small experiment

Take N matches of length L.

- Draw equi-spaced parallel lines on the floor by cleverly choosing the distance $2L$. Randomly throw the matches on the floor.
- Count the matches which cross or touch the lines, say those are M.
- What you eventually get when you divide N by M is truly striking!

$$\frac{N}{M} \cong \pi$$

Featured articles

It's simple yet not easy

Dr. Md Ibrahim Molla

"Theorems are fun especially when you are the prover, but then the pleasure fades. What keeps us going are the unsolved problems."

- Carl Pomerance

You are probably wondering how a simple thing cannot be easy, right? But trust me, it is true! By the end of this article, you too will feel the same way, thinking, 'Ugh, it's really simple buuut…'

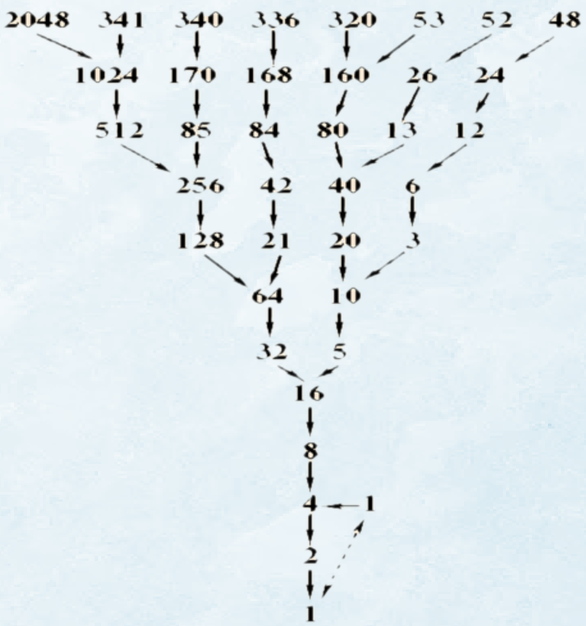
We know that mathematics plays a vital role for human development that has constantly been extending its bounds of knowledge and capability. On one side, it is replete with elegant theories and irrefutable proofs, on the other side, it harbours a collection of tantalizing enigmas open problems. These mathematical problems naturally captivate the minds of mathematicians and hold the potential to unlock the new boundaries that would enhance our understanding of the universe.

Here we embark on a journey of some mind-blowing conjectures whose solutions require innovative approaches and groundbreaking insights. *Who knows? You might just be the one to crack one of these problems which propels you to the heights of recognition with a Fields Medal, Abel Prize, Infosys Prize, Bhatnagar Award, or any other esteemed honor.* Okay, let us begin with the simplest open problem.

❖ **The Collatz Conjecture:** You will surely be tempted by the simplicity of this problem. Just choose a (whole) number, any number of your choice. If the number is even, halve it; if it is odd, triple it and add 1. Take the resulting number and repeat the process. Now the fascinating fact is that if you keep on going, you will eventually get stuck in a loop.

For example, take the number 5: As 5 is odd, we triple it and add 1 to get 16, which is even. So, we halve it to get 8, then halve that to get 4, then halve that again to get 2 and then halve 2 once more to get 1. Now, since 1 is odd, we triple it and add 1. Here we are back at 4! And we know how this number 4 goes: 4 goes to 2, which goes to 1, which goes back to 4 again and so on. Oh, here we are stuck in a loop! Fascinating, isn't it?

There is nothing special about the number 5. In fact, if you try this process with any other positive integer, you will always fall in this loop and the Collatz Conjecture claims that *no matter which number (positive integer, of course) you start with, the sequence will inevitably lead you into the same endless loop.*



You have probably noticed just how simple this problem is - it is so simple that one can easily explain it to a primary-school-aged kid, and they would likely be intrigued enough to try and find the answer on their own. Yet, its charm lies in its deceptive simplicity, as it has fascinated generations of mathematicians, drawing them into its mystery and sparking countless attempts to unravel its secrets.

Although no one has been able to completely prove this conjecture till today, it has been verified for every number less than 2^{68}. A few years back, in 2019, Terence Tao, a Fields medalist and one of the world's top living mathematicians, made significant progress by proving that the Collatz Conjecture is *almost* true for *almost all* numbers but the complete proof of this enigmatic conjecture is still open.

❖ **The Erdős–Straus conjecture:** Here is another mind-blowing problem.
But before diving into it, let us quickly review some key terminologies. A positive fraction is said to be a *unit fraction* if its numerator is 1. For example, 1/1, 1/2, 1/7 are all unit fractions. As a warm-up exercise (please verify!), it is interesting to check that every positive rational number can be expressed as a sum of distinct unit fractions. A finite sum of distinct unit fractions is known as an *Egyptian fraction*, a concept that dates to ancient Egyptian mathematics. So, the previous exercise can be rephrased as: every positive rational number is an Egyptian fraction.

Now, let us turn our attention to the positive rational numbers of the form $\frac{4}{n}$, where $n > 1$. In 1948, Erdős and Straus conjectured that every positive rational number of this form can be expressed as a sum of *exactly three*-unit fractions. In other words, for any positive integer $n > 1$, there exist positive integers x, y and z such that the equation

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

holds true. For example:

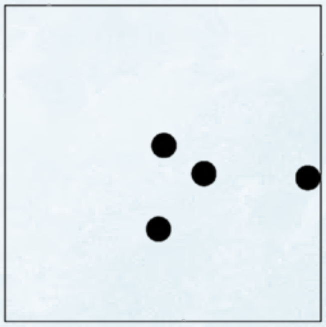
- $\frac{4}{2} = 2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{1}$,
- $\frac{5}{3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{6}$,
- $\frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20}$ etc.

Till date, the truth of this seemingly innocent conjecture has been verified for all n up to 10^{17}, without any single exception but despite this extensive computational evidence, no general proof has been found.

❖ **Happy Ending Problem:** As we approach the end of this article, it seems appropriate to discuss the intriguing *Happy Ending Problem*.

We know that for any positive integer $n \geq 3$, an n-gon is considered to be convex if each of its n corners is less than 180°. Now the problem is: for a positive integer $n \geq 3$, what is the smallest number $h(n)$ of points such that any general arrangement (i.e., no three points are collinear) of $h(n)$ points in a plane must contain at least a set of n points that form the vertices of a convex n-gon. This intriguing problem, known as the *Happy Ending Problem*, was named by the renowned mathematician Paul Erdős. *The problem was named so as it led to the marriage of his two students, George Szekeres and Esther Klein, who were the first to work on it.*

Since every triangle is convex, it is obvious that $h(3) = 3$. Now let us consider the following 4 points:



A simple observation asserts that no convex quadrilateral can be formed from these 4 points, implying that $h(4) > 4$. In fact, it was proved by E. Klein that $h(4) = 5$. Therefore, it is always possible to form a convex quadrilateral from any given set of five random points in a plane.

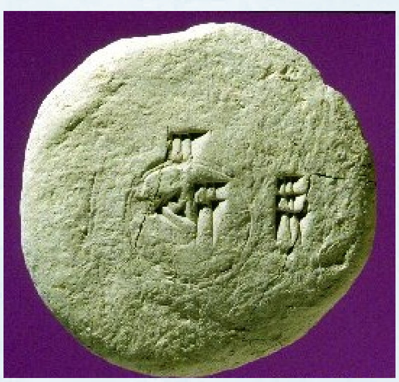
As of now, the values of $h(n)$ are known only for $n = 3, 4, 5$ and 6 which are 3, 5, 9 and 17 respectively and determining the value of $h(n)$ for $n > 6$ remains as a wide-open problem. $h(5) = 9$ was proved by E. Makai in 1970 and more than 3 decades later, in 2006, Szekeres and Peter proved that $h(6) = 17$. Based on the known results, it was conjectured by Erdős and Szekeres that $h(n) = 1 + 2^{n-2}$, for all $n \geq 3$. They could also prove that $h(n) \geq 1 + 2^{n-2}$, for all $n \geq 3$. So, the only remaining question for this conjecture is to check if the reverse inequality is equally valid or not.

These are some innocent and simple looking open problems each of which can be easily explained to a *high-school level* (you may recognize this terminology used by the famous Indian American mathematician S. S. Abhyankar) student but all of them remain unsolved for many decades. These problems are simple in the sense that they are simple enough to understand and explain but their solutions are far from easy (perhaps, it is time to look back at the title of this article again; ugh...). *But these are not at all useless, maybe one of you reading this article will make the breakthrough necessary to solve one of these challenges and cement your place in history.*

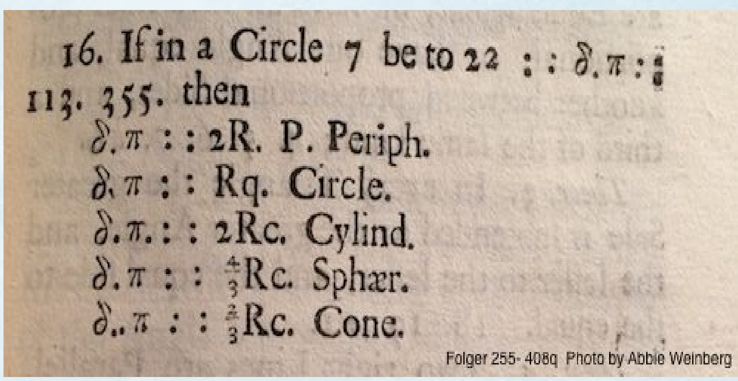
Chasing π: A Journey Through the History and Mystery

Timir Karmakar

The remarkable journey to uncover the value of π is a captivating tale that stretches across thousands of years, showcasing the brilliance of mathematicians. Throughout this journey, mathematicians have made significant effort in enhancing their understanding of π. One of the important aspects in this regard was calculating its value to a high degree of accuracy. This involves the contribution of mathematicians from various cultures around the world. The earliest documented calculations of π date back to the Babylonians, Egyptians and Hebrew civilizations, all of which played a crucial role in advancing the understanding of this fundamental mathematical constants. The ratio of circumference of a circle to its diameter equal to 3 even appears in the verse of Hebrew Bible which written around 10th or 11th BCE proving that 3 was considered as an approximation of what we know today as π. A verse of Hebrew Bible reads (see Chronicles 4:2, King James Version)



A Babylonian Clay Tablet depicting value π (Source: Yale Babylonian collection)



16. If in a Circle 7 be to 22 : : δ.π : : 113. 355. then
δ.π : : 2R. P. Periph.
δ.π : : Rq. Circle.
δ.π. : : 2Rc. Cylind.
δ.π : : $\frac{4}{3}$Rc. Sphær.
δ..π : : $\frac{2}{3}$Rc. Cone.

Folger 255- 408q Photo by Abbie Weinberg

This image is taken from the 1694 edition, the text is unchanged from the 1647 printing[/note].

"Also, he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about." This refers to a large, bronze basin built for the temple in Jerusalem by King Solomon with diameter of ten cubits and a circumference of thirty cubits, suggesting that it is circular if π is approximated to 3. One Babylonian clay tablet dated 1900-1680 BCE and unearthed in 1936 at Susa, 200 miles away from Babylon indicates a value of 3.125 for π, a closer approximation. The *Rhind Papyrus,* an Egyptian document dating from *circa* 1650 BCE states that the area of a circular field of diameter 9 units is equal to the area of a square with side 8 units, this leads to approximate value of 3.1605 for π. The first theoretical calculation regarding the approximate value of π was done by Archimedes of Syracuse (287-212 BCE) based on polygon approach who showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$. Around 150 AD, the Greek Roman scientist Ptolemy, in his work *Almagest*, provided a value of π as 3.1416, which he likely derived from the limits set by Archimedes or *Apollonius of Perga.* Zu Chongzhi an esteemed Chinese mathematician and astronomer, used a similar approach to determine the value of π. Although he likely wasn't familiar with Archimedes' technique, much of his work remains unclear due to the loss of his book. He determined that π lies between 3.1414926 and 3.1415927, making him the first scientist in the world to calculate the value of π up to seven decimal places, a record precision which dominated almost 900 years. The Hindu mathematician Aryabhata in his work *Ganitpada* discusses π in the following verse (Ganitpada, 2.10)

“Caturadhikam satmastgunam dvastistatha sahastranam

Ayutdvayavishlambhasayasanno vruttaparinah’’

According to this verse if the diameter of a circle is 20000, its circumference is about 62832 which gives π is approximately equals to 3.1416. Another Hindu mathematician and astronomer, Brahmagupta put forward the concept that the value of π is approximately equals to the square root of 10. Bhaskaracharya an Indian mathematician and astronomer, in his work Lilavati (verse 199), calculated the approximate value of π as 3.141666. In the 16th and 17th centuries, the calculation of π underwent a revolution with the introduction of infinite series techniques, especially in Europe. These methods allowed for much greater precision in determining π values. The French mathematician François Viète gave a formula to determine the value of π also known as Viète’s formula. He was the first mathematician to represent π as an infinite product. Using his formula Viète calculated the value of π to an accuracy of nine decimal places. Ludolph van Ceulen (1540-1610) a German Dutch mathematician devoted a great part of his life working on π. He calculated the value of π to an accuracy of 35 decimal places. In Germany, π is still sometimes referred to as “Ludophile number” remembering his contribution in determining approximate value of π. Up until this time, no symbol existed to represent the ratio of a circle’s circumference to its diameter. In 1647, William Oughtred (1574-1660), in his book *Clavis Mathematicae* first time introduced the symbol π to represent the circumference of a circle with its value change on circle’s diameter, rather than representing the constant value of π that we recognize today. In this book Oughtred used π to represent the periphery of a circle and δ to represent diameter in the ratio. In those times, the circumference of a circle was referred to as the ‘periphery,’ and the Greek letter ‘π’ the equivalent of ‘p,’ was used to denote it. In 1706 William Jones (1675-1749) a Welsh mathematician first time used the symbol π to represent the ratio of circumference of a circle to its diameter in his book *Synopsis Palmariorum Matheseos.* The symbol for π was not widely adopted at first, but it gained universal acceptance in 1737 when Leonhard Euler began using it. In 1873, the Englishman William Shanks calculated the approximate value of π correct up to 707 decimal digits which was initially correct up to 527 decimal digits. Later, Daniel F Ferguson (1841-1916) discovered the error in Shanks calculation and raised the accuracy up to 808 decimal digits. Shanks's approximation remained the longest known expansion of π until the advent of digital electronic computers in the 1940s.

With the advancement of computers, mathematicians made significant efforts to calculate increasingly more digits of π. In 1949, a significant breakthrough took place, not in mathematical theory, but in the speed with which calculations could be carried out. After the invention of Electronic Numerical Integrator and Computer (ENIAC) in 1945, a team lead by George Reitwiesner and John von Neumann used the ENIAC computer to calculate the decimal expansion of π up to 2035 places just in 70 hours. In 1961, John Wrench and Daniel Shanks calculated 100,265 digits of π using the IBM 7090, and by 1973, one-million mark was surpassed by Jean Guilloud and Martine Bouyer. In 1997, Kanada and Takahashi calculated 51.5 billion digits of pi. Two years later, in 1999, they increased the record to 68,719,470,000 digits. Then, in 2010, Yee and Kondo broke new ground by calculating 5 trillion digits. Since then, the calculation of π has evolved chronologically, achieving increasingly higher precision. As of June 28, 2024, the current record for calculating pi stands at 202,112,290,000,000 digits. This record was set using y-Cruncher v0.8.3 by Jordan Ranous, Kevin O'Brien, and Brian Beeler.

The quest for π continues, with scientists actively searching for patterns within its digits and pushing the boundaries of precision through the development of new algorithms. As a result, we can conclude that even more surprises remain hidden in the depths of undiscovered knowledge about this famous constant. We eagerly anticipate what the future will unveil in this ongoing pursuit.

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Archimedes: His Life and Legacy

Dibyasman Sarma, Research Scholar

There is geometry in the humming of the strings, there is music in spacing of the spheres.

– Pythagoras

Archimedes of Syracuse (c. 287–212 BCE) is best remembered for jumping out of his bath and running unclothed through the streets shouting, “Eureka! Eureka!” (“I have found it!”), when he discovered how to distinguish a genuine gold crown from a counterfeit crown. It is less well known that he devised a test to distinguish the greatest mathematician of antiquity, himself, from impostors. Mathematicians in the Greek realm of mathematics were known to send out announcements of their newly discovered mathematical theorems without providing proofs. This was a common practice. When Archimedes suspected others of claiming his results as their own, he inserted two or three propositions into his own announcements that required all his mathematical prowess to demonstrate to be false, and he exposed them by sending counterexamples when they asserted the false statements as their own newly discovered truths.

Early Life and Education

Archimedes was the son of Phidias, an astronomer, and was born in the ancient Greek colony of Syracuse, which was located on the island of Sicily. Little is known of his early education, but it is believed that he studied in Alexandria, Egypt, which was a well-known center of learning in the Hellenistic period. Archimedes was well-acquainted with the work of earlier Greek mathematicians, and his genius was apparent early on. His commitment to understanding the natural world and solving mathematical problems set him apart from his peers.

Major Contributions to Mathematics

Archimedes' contributions to mathematics are extensive, although one of his most renowned accomplishments is the development of the *"method of exhaustion"*, sometimes called *"indirect passage to the limit"*, an early precursor of integral calculus. He employed this technique to compute the areas and volumes of irregular shapes. This mathematical invention established the foundation for further advancements in calculus, centuries before Isaac Newton and Gottfried Wilhelm Leibniz would formally develop the discipline.

His study on the measuring of the circle comprises three propositions, and the second one in order, which compares the area of a circle to a square on its diameter, depends on the third proposition which states that the ratio of a circle's circumference to its diameter is greater than $3\frac{1}{7}$ but less than $3\frac{10}{71}$, a fairly good approximation of π. It is the first proposition that holds the most interest for us. It states that the area of a circle is equal to the area of a right triangle in which one of the sides about the right angle is equal to the radius and the other is equal to the circumference. Notice the interesting way in which Archimedes' statement of the theorem equates an area bounded by a curve, the circle, with an area bounded by straight lines, the right triangle. (From a modern perspective, we express the area of the circle as πr^2. In modern notation Archimedes expresses it as $\frac{1}{2}(2\pi r)r$).

Another key accomplishment in mathematics was his work on the geometry of spheres, cylinders, and cones. Archimedes proved that the volume of a sphere is two-thirds of the volume of the cylinder that contains it, and he discovered the surface area of a sphere as well. His formula for the volume and surface area of a sphere remains central to modern geometry.

Legacy and Influence

When King Hieron challenged him to move a great weight with a small force, Archimedes conceived the idea of the compound pulley and showed how he could easily pull in to shore a three-masted ship that 100 men could only pull in with much difficulty. According to the ancient Roman biographer Plutarch, it is in connection with this story that Archimedes uttered his famous remark, "Give me a place to stand on, and I will move the earth." His habit of ignoring his surroundings would end up costing him his life. Archimedes' engineering accomplishments building war engines made him a prime target of the Roman army that invaded Sicily in 287 B.C., during the Second Punic War. Legend records that the Roman soldier found Archimedes drawing figures in the sand. The soldier commanded Archimedes to stop what he was doing and leave immediately. Archimedes asked for more time to work out a problem in the sand. Enraged, the soldier ruined Archimedes' figures in the sand and ran him through with his sword!

Though Archimedes' life was cut short, his work had a lasting impact. His writing influenced generations of mathematicians, scientists, and engineers throughout antiquity and into the Renaissance. Figures like Leonardo da Vinci and Isaac Newton were inspired by Archimedes' methods and inventions. His discoveries, particularly in the areas of geometry, mechanics, and fluid dynamics, laid the groundwork for later advancements in science and engineering. Today, Archimedes is celebrated as a polymath whose contributions transcend both time and discipline. From the use of his principles in engineering projects to the continued study of his mathematical theories, Archimedes' legacy lives on in the very fabric of modern science and technology.

Conclusion

Archimedes was an individual whose intelligence and imagination were remarkably advanced for his era. His findings not only contributed to the advancement of scientific knowledge within his time period, but they also laid the groundwork for subsequent generations to build upon. Archimedes' contributions to knowledge and technology continue to inspire and resound throughout the world of science and engineering. These contributions might be attributed to his mathematical brilliance, his inventions, or his insights into the physical universe.

Source: *God Created the Integers: The Mathematical Breakthroughs that Changed History, Stephen Hawking*.

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