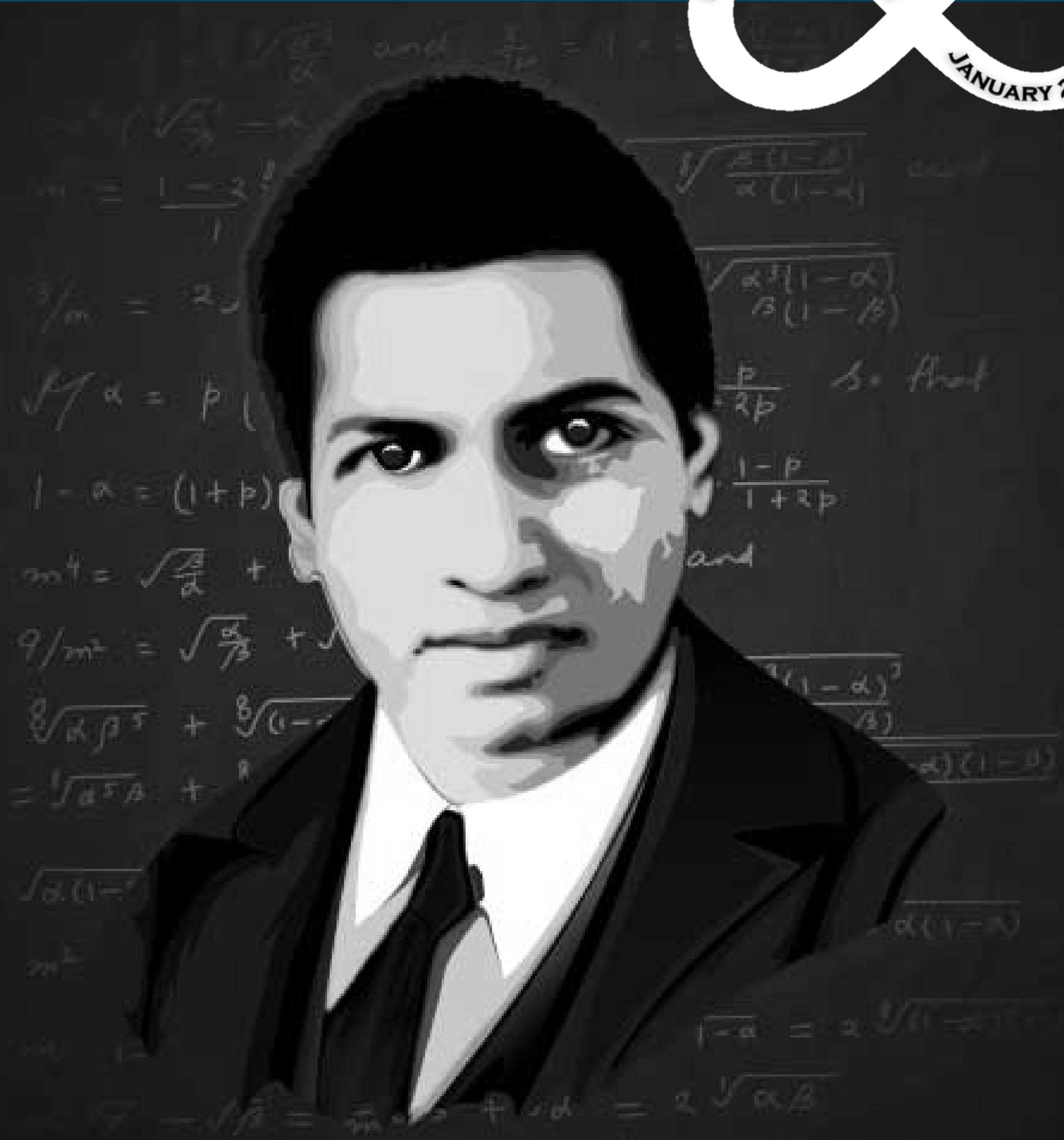
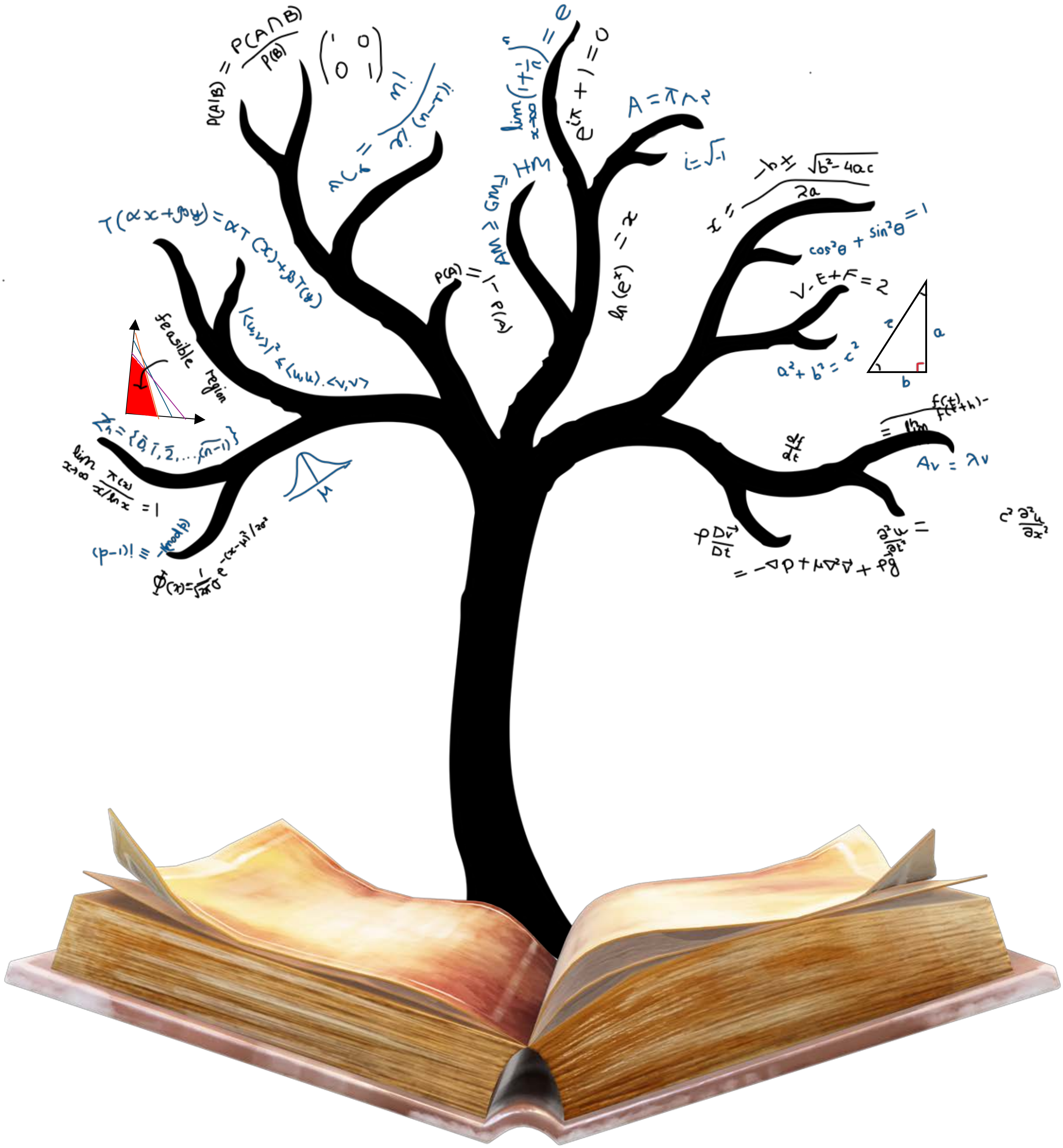
NITM Mathematical Bi-monthly

DECEMBER 2024 ∞ JANUARY 2025



Department of Mathematics,
National Institute of Technology Meghalaya
Saitsohpen, Sohra (Cherrapunji)-793108, India



feasible region
A = πr²
i = √-1
x = -b ± √(b²-4ac) / 2a
cos²θ + sin²θ = 1
V - E + F = 2
a² + b² = c²
Av = λv
e^{iπ} + 1 = 0
AM ≥ GM ≥ HM

“An equation has no meaning for me unless it expresses a thought of God.”

— Srinivasa Ramanujan Aiyangar

(22 December 1887 – 26 April 1920)

DIRECTOR'S MESSAGE

Dear Students, Faculty, and Readers,

I am immensely pleased to introduce the 3rd edition of the Department of Mathematics' bimonthly magazine. This magazine represents a significant step forward in creating a platform where the department can showcase our students and faculty members' intellectual curiosity, talent, and dedication.

Mathematics is not just a subject confined to classrooms and textbooks; it is a dynamic and evolving field with the power to shape the world around us. I am proud of the department's commitment to fostering academic excellence and a spirit of innovation.

This magazine is a testament to that pursuit of knowledge. It will serve as a medium for not only disseminating new ideas and research but also for encouraging discussions, collaborations, and creativity within our vibrant mathematical community. I encourage each of you—students and faculty alike—to contribute actively to the growth of this magazine and make it a reflection of our collective brilliance.

As we move forward, let us continue to strive for academic distinction, intellectual curiosity, and a passion for solving the complex problems that mathematics presents. The journey is as important as the destination. I believe that together, we will continue to make strides toward a brighter future for the department and the world of mathematics.

I congratulate the editorial team on their hard work in bringing this publication to life, and I look forward to seeing the magazine evolve in the years to come.

With best wishes,

Prof. Pinakeswar Mahanta

Director, NITM

HoD’s Message

As the Head of the Department of Mathematics (MA), it is truly an honor for me to write for the 3rd edition of our departmental magazine. This platform allows our faculty, staff, and students to showcase their achievements, express their opinions, and explore new mathematical concepts. I believe that this magazine will become an essential channel for sharing knowledge, ideas, and research insights that will inspire us all.

The Department of Mathematics, which started functioning in June 2012, currently offers 2-year M.Sc. and PhD programs. M.Sc. students are selected based on their ranking through CCMN and the institution mode, while PhD students are selected through interviews based on their GATE/NET scores. In addition to our core programs, the department also plays a vital role as a supporting pillar for various B. Tech and M. Tech programs within the institute.

The creation of this magazine stems from a collective desire to share our thoughts, accomplishments, and aspirations. Working together as a team to ensure its successful publication brings immense delight. I feel privileged to be part of this process and am filled with joy in nurturing our students, contributing to society, and fostering academic excellence.

My team and I remain dedicated to the holistic development of our students within the institute. I extend my best wishes to all MA family members and sincerely hope that this tradition of the departmental magazine continues for generations to come, fostering happiness, unity, and intellectual growth.

Warm regards,
Dr. Adarsha Kumar Jena
Assistant Professor, HoD, MA

Editor’s Message

The only way to learn mathematics is to do mathematics. — Paul R. Halmos.

This profound statement not only serves as a guiding principle but also emphasizes the importance of active engagement in mathematics. It brings me great joy to inform you that the Department of Mathematics at the National Institute of Technology Meghalaya is introducing its 3rd edition of its own publication, the “*NITM Mathematical Bi-monthly.*”

This publication is a collective endeavor by our students and faculty members, designed to ignite a love for mathematics and offer a stage for students to share their insights. Magazines transform the creative potential of our students into tangible contributions, allowing them to identify and showcase their talents through writing. Through this magazine, we aspire to highlight contributions, departmental events, achievements, and the scholarly work of both faculty and students. I encourage all students to participate by submitting interesting mathematical problems, engaging puzzles, stories, and intriguing facts about mathematicians.

I want to express my deepest appreciation to the editorial team—Bankit, Sanchita, Dixita, and Dibyasman—for their tremendous dedication and hard work in making this magazine a reality in such a brief period. Our minds are filled with boundless curiosity, and we are continually striving to explore beyond the known. I wish all our students’ immense success as they delve into the magazine's contents and set out on fresh intellectual journeys. May this initiative inspire us all to deepen our grasp of mathematics with steadfast determination.

Thank you, and best wishes.

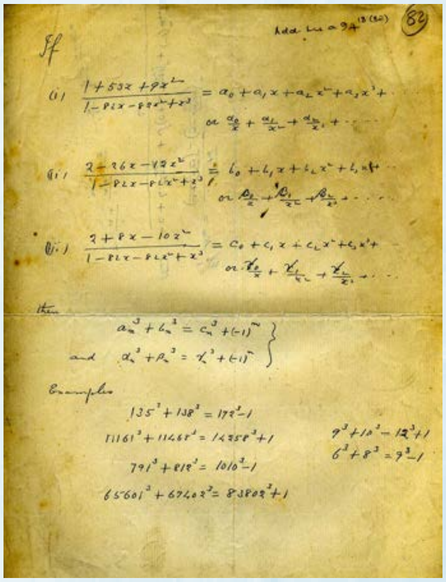
Dr. Timir Karmakar
Assistant Professor
Department of Mathematics

The Lost Notebook of Ramanujan: A Pivotal Content of Mathematical Mysteries and Exploration

Bankitdor M. Nongrum, Research Scholar

Srinivasa Ramanujan Aiyangar was one of the most brilliant and enigmatic mathematicians of the 20th century. He has left behind a legacy that continues to captivate all academicians and mathematicians alike. The Government of India has declared his birth anniversary (22 December) as the National Mathematics Day to be celebrated annually. Ramanujan was born in Erode, in the then colonial India, but he has tragically lived a brief life-he passed away at the age of 32 in 1920. He has made groundbreaking contributions to mathematics, including number theory, infinite series, and continued fractions, despite having not received any formal education in the field. His "Lost Notebook," is a collection of unpublished manuscripts that were rediscovered decades after his death, is one of his most compelling legacies.

The Lost Notebook of Ramanujan was neither a typical "notebook," nor was it "lost" in the traditional sense of having been misplaced during his lifetime. Rather, it was a collection of papers that he worked on in the final year of his life, specifically between 1919 and 1920, while he was battling tuberculosis in India after returning from England. The papers were sent to mathematicians in England following his passing; however, they were mostly ignored. Hundreds of unpublished results that Ramanujan had compiled are included in them, many of which were previously unknown to the mathematical community. They were uncovered in the archives of the Trinity College Library in Cambridge by the renowned mathematician George Andrews in the spring of 1976. Andrews discovered a wealth of information, including over 600 formulas and results that were beyond the mathematical knowledge of Ramanujan's day. They have inspired research for almost a century and are brimming with groundbreaking concepts, mysterious equations, and unproven theorems.



A popular page from the notebook

What are the contents of this lost notebook?

The Lost Notebooks consist of several key themes in mathematics. These include:

1. q-Series and Continued Fractions: A q-series is a sequence of functions that is defined by a particular kind of power series. Ramanujan's fascination with these series established some foundations for string

theory, modular forms, and combinatorics. Additionally, Ramanujan demonstrated exceptional excellence in the field of continued fractions, which involved an extensive number of identities. Many of these were stated without proof, requiring mathematicians to verify or expand on them, a work that is still ongoing to this day.

2. Modular Forms and Group Theory: Ramanujan’s work in the theory of modular forms and group theory was groundbreaking. Many of the results contained in the notebooks were ahead of their time and continue to influence contemporary research in mathematics.

3. Mock Theta Functions: The most remarkable discoveries in the notebook were Ramanujan's investigations into "mock theta functions." Ramanujan had actually introduced them in a letter to G.H. Hardy shortly prior to his passing. These functions are a generalization of the classical theta functions in the theory of modular forms, and they had not been studied by mathematicians at the time. Mock theta functions are intricately linked to the theory of modular forms and partitions and are defined by specific q-series. Ramanujan's research on these functions was largely overlooked for many years, but it subsequently became a central focus of the theory of modular forms and the theory of q-series. Mathematicians were puzzled by their significance for decades; however, they have since been associated with contemporary disciplines such as black hole physics and quantum mechanics.

4. Ramanujan's Conjectures: Additionally, the manuscript contained numerous conjectures that Ramanujan had proposed without providing any proof. Several of these conjectures were subsequently verified, while others were the focus of substantial mathematical investigation in the years that followed the notebook's discovery. The conjectures addressed a diverse range of subjects, such as the asymptotic behaviour of partition functions and the development of novel classes of infinite series.

Challenges and Mysteries

Decoding the notebook is no easy task. Ramanujan's thoughts were frequently condensed into brief words, his handwriting was occasionally unreadable, and his notation was odd. For example, a theorem that requires pages for explanation could be implied with a single paragraph. Verification is made more difficult by the lack of proofs; some claims have taken decades to be solved, while others are difficult to resolve. Given that Ramanujan wrote on whatever paper he could find, some of which may not have survived, the incomplete nature of the notes also begs the question of what might have been lost.

How Ramanujan arrived at his conclusions is one unsolved puzzle. His preference for intuition over explanation indicates a poorly understood cognitive function. Did he have a near-mystical ability to understand mathematical facts, or did he notice patterns that others couldn't see? This mystery enhances his appeal, turning the Lost Notebook into a question of philosophy as well as a scientific treasure.

Mathematical Impact

In the 20th and 21st century, mathematicians grew interested in the mock theta functions, and in the decades following their discovery, significant advancements in our knowledge of these functions occurred. The study of number theory was profoundly transformed by the discovery of the Lost Notebook, and in other fields like partition theory, modular shapes, and q-series. Studying Ramanujan's findings in these notebooks has produced fresh perspectives and links to a number of mathematical disciplines, including as algebra, combinatorics, and mathematical physics.

Additionally, the notebooks have had a significant impact on the advancement of contemporary computational methods. Algorithms used in computer mathematics, specifically for computing huge numbers, partition functions, and modular forms, incorporate Ramanujan's identities and series.

Srinivasa Ramanujan’s Legacy

The legacy of Ramanujan has crossed beyond the limits of discoveries found in his Lost Notebook. From his innate sense of intuition, to his ability to produce results without proofs, and his deep understanding of number theory have inspired generations of mathematicians. Despite a brief life on this world, the influence on mathematics he left behind is infinite, and the pun is intended. The notebook provides a glimpse inside Ramanujan's thoughts, showing how his work developed and how he came up with some of the most significant mathematical discoveries ever made.

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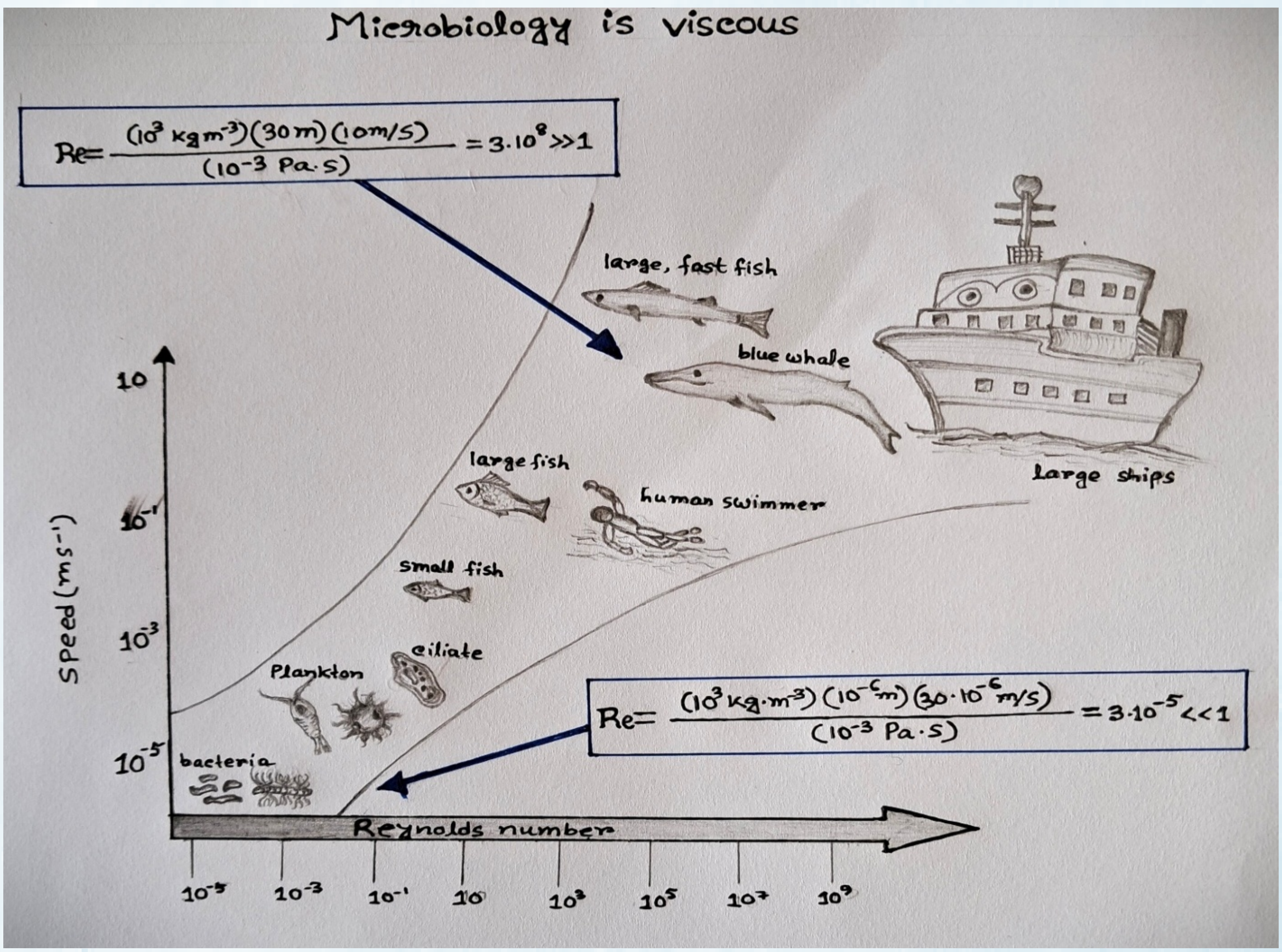
Thriving at Low Reynolds Numbers: The Science of Life in a Fluid World

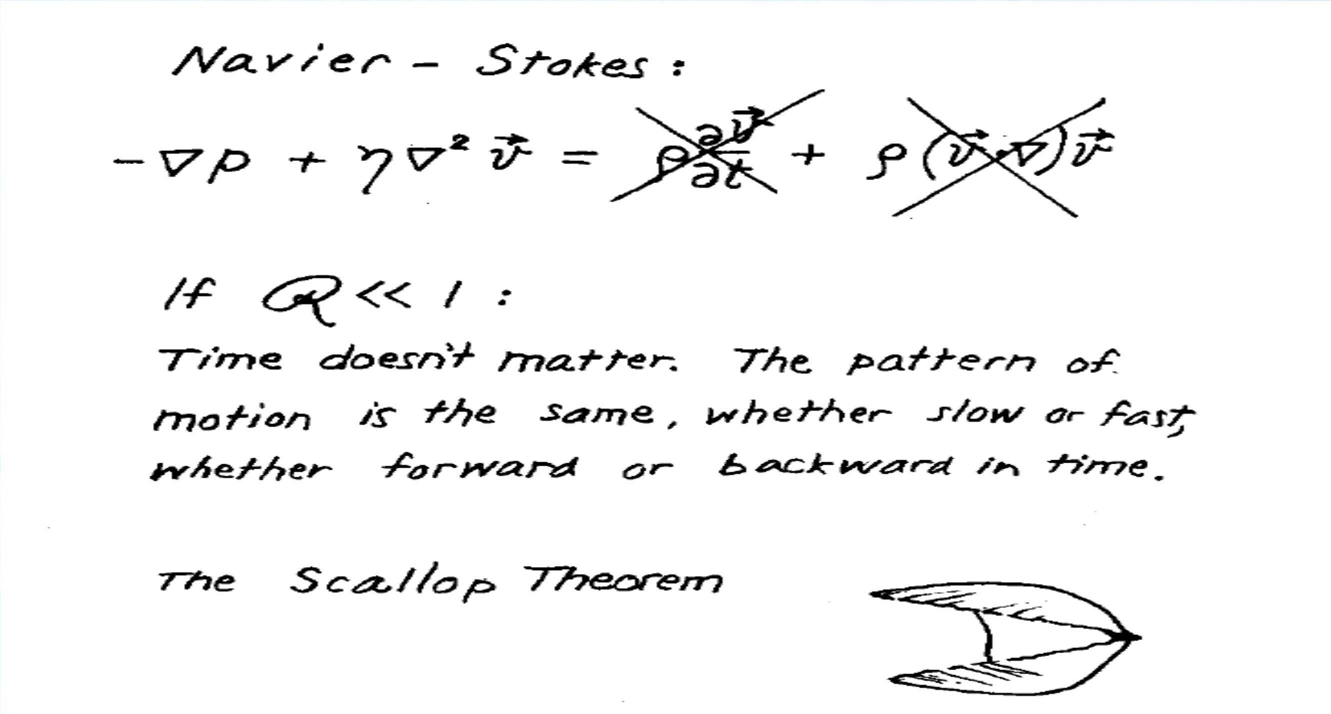
Timir Karmakar, Sanchita Pramanik

Have you ever been curious about how microorganisms manage to live in just a single drop of water? Why does water appear solid to certain creatures, and how does this impact their behaviours? Let's try to explain answer to these questions in details. The content of this article is inspired by the talk "**Life at Low Reynolds Number**," delivered by Nobel laureate and American physicist E. M. Purcell, which was later published as a paper in June 1976. Motility is so important for the survival of many organisms. For a tiny life such as bacteria and other microorganisms' locomotion offers unique challenges that differs significantly from those experienced by a macroscopic organism.



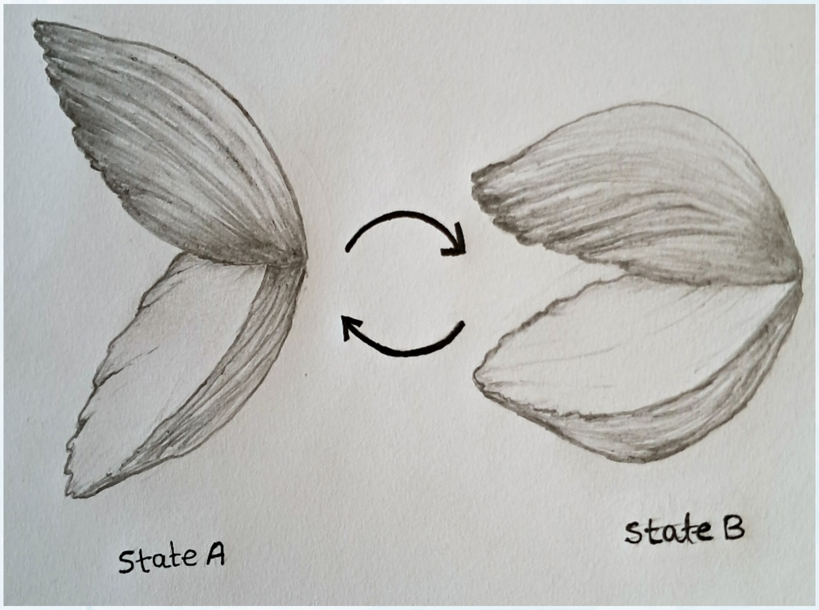
Most microorganisms inhabit fluid environment where they encounter a viscous force that is several order magnitudes greater than inertial forces. This is where life becomes remarkably exciting for species and systems that operate on the microscale. This world is quite different from the one that we are living. Perhaps the most notable exploration of this world was carried out by physicist E.M. Purcell in his lecture '**Life at Low Reynolds Number**.' This is due to the low Reynolds number (Re) which is the ratio of the inertial and viscous forces for an object moving through a fluid. It can be thought as a measure of how effectively an object moves through a specific fluid at a given velocity. $Re = \frac{inertial\ forces}{viscous\ forces} = \frac{\rho VL}{\mu}$. In this formula ρ is the fluid density, V is the object velocity, L its significant dimension and μ is the kinematic viscosity.

For a person swimming in water Re could be in the order of 10^4, while for a fish is order of 10^2. This is one of the reasons why fish are so difficult to catch in bare hands. Water feels denser to them than it does to you. At the microscopic level, such as for organisms like paramecia or bacteria, the Reynolds number of water can fall as low as 10^{-4}. In this low Reynolds number scenarios, the influence of inertia becomes minor, and the motion dynamics are largely dictated by viscous forces. At such low Reynolds number, the only forces that influence a body are those acting on it currently. Previous motion has no effect, and even the concept of time becomes insignificant. Thus, instead of focusing on forces over a period of time, the behaviour of a microorganism is mainly shaped by the immediate forces acting upon it. A consequence of this is the '**Scallop Theorem'** which was introduced by Purcell in his 1976 paper on 'Life at low Reynolds number'.



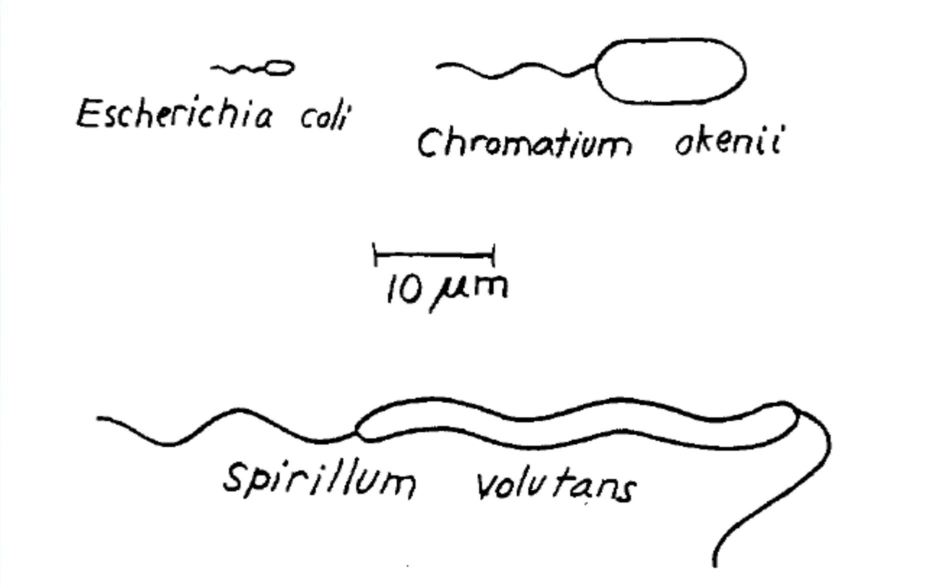
'***Life at Low Reynolds Number'*** by E.M. Purcell, American Journal of Physics, Vol 45, No 1, January 1977.

Having a low Reynolds number carries profound implications, because it indicates that viscous forces dominate the system, while inertial forces become negligible. Let's think how a person feels swimming in water. When swimming, a person propels themselves forward by pushing against the water, creating a momentum. As they move ahead, they use their arms and legs to reposition for the next stroke. This reciprocal motion—pulling their limbs back into position—naturally slows them down, but the momentum they built up from the previous stroke helps them maintain speed. With each subsequent stroke, they regain and even build on that momentum, allowing them to keep moving forward efficiently despite the retardation between strokes. This scenario is completely different in a low Reynolds number environment. You cannot gain the momentum as you swim through a viscous fluid having a low Reynolds number. When a swimmer executes a reciprocal motion in a low Reynolds number environment, i.e., following a sequence of shape changes that are identical when reversed, the net displacement traverse by the swimmer must be zero, if the fluid is incompressible and Newtonian. Purcell beautifully mentioned this in the name of "**Scallop Theorem**" borrowing from the name of one of such organisms- a scallop which cannot locomote in absence of inertial force. In Purcell's own words, 'So, if the animal tries to swim by a reciprocal motion, it *can't go anywhere*. Fast or slow, it exactly retraces its trajectory and it's back where it started. A good example of that is a scallop. You know, a scallop opens it shells slowly and closes its shell fast, squirting out water. The moral of this is that the scallop at low Reynolds number is no good. It can't swim because it has only one hinge, and if you have only one degree of freedom in configuration space, you are bound to make reciprocal motion. There is nothing else you can do.'



A scallop swims by slowly opening its two halves, then quickly closing them. This is a successful swimming method because the inertial force of the surrounding water dominates over the viscous force. In a low Reynolds number environment, a scallop would only oscillate in place.

To solve this problem, locomotion at low Reynolds numbers typically necessitates nonreciprocal motion for the swimmer. In nature, microorganisms break time-reversal symmetry through rotating helices and cilia that exhibit flexible, oar-like beats. While the corkscrew works similarly to a ship's propeller, the flexible oar works well because it bends in a way that keeps it from pushing in the opposite direction during recoil. Despite their complexity, propellers make sense to us intuitively, but Purcell states that this is only a coincidence since it doesn't accurately capture the underlying dynamics. Bacteria such as E. Coli utilizes its flagellum in a corkscrew motion while moving in low Reynolds number environment.



Source: '***Life at Low Reynolds Number***' by E.M. Purcell

But why should we worry about this tiny world? The answer to this question leads us to its enormous application in the field of microbiology and medical science. Scientists are replicating microbial swimming to create small robots for targeted medicine administration, which are small devices that can navigate the bloodstream with efficiency. The way this microorganisms locomote give valuable insight into life-saving technologies. So, the next time you mix your coffee, see how the liquid twists and spins, then imagine a microbe in the same cup, straining against large amount of resistance. In their small universe, water is more than simply a gentle flow; it is a constant force that dictates their every activity. Nonetheless, they flourish, utilizing creativity of nature to traverse a world of sluggish, steady perseverance.

Purcell's remarks may appear simple, but they hold immense significance. They establish the groundwork for a strictly geometrical perspective on cell locomotion and have attracted considerable interest in the bio- locomotion field from both physics and soft matter researchers. Indeed, “**Life at Low Reynolds Number**” has emerged as the most referenced article in this domain.

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Revolutionizing Medicine and Healthcare with Nanotechnology

Dixita Sonowal, Research Scholar

Nowadays, nanotechnology—a phenomena that takes place at nanoscale dimensions—is thought to be the most potential technological advancement of the twenty-first century. Richard Feynman, a physicist, initially proposed the idea of nanotechnology in 1959 during a talk on creating things at the cellular and molecular level. The application of nanomedicine in the medical field has already greatly improved medication. Certain nanoparticles have shown great promise in tissue engineering, pharmaceutical products, targeted medical products, diagnostic tools, chemotherapeutic cancer medications, and other areas.

Numerous classes of nanomaterials, including metal nanoparticles, carbon nanomaterials, magnetic nanomaterials, semiconductor nanomaterials, polymer nanocomposites, biodegradable polymers, and many more, have been reported in the health care sector. These methods have shown encouraging results and significant advancements.

The various associated features and characteristics of nanotechnology in medical science include:

1. **Minimise damage of healthy cells in cancer treatment**: Through the usage of nanotechnology, doctors can target the tumour cells more cautiously by magnetic nanofluids without harming neighbouring good tissues. Physicians can also provide heat treatment with more accuracy utilizing nanotechnology along with cancer therapy.
2. **Nanomedicine diagnostic technique**: Usage of carbon nanotubes, application of gold nanoparticles made illness identification feasible at an earlier stage.
3. **Wond treatment**: Use of gold nanoparticles and quantum dots, employing infrared light, improved cleaning of instruments and could be used for antibacterial treatments. Manipulation of an item to produce compounds which have extraordinarily diverse and unique characteristics at the atomic and molecular levels are done by nanotechnology and can be used for various disease treatments.
4. **Antibacterial treatment**: Research into this subject, supported by the knowledge of nanoscale tissues, attempts to construct tissues whose function and structure are similar to those found natural to the organism. Nanogenerators and polymer nanoparticles could help people’s wound to get better and treated quickly.
5. **Drug delivery**: Iron-based nanoparticles are used as delivery vehicles of drugs for cancer patients.
6. **Tissue engineering**: Nanomaterials such as nanofibers and nanocomposites can be used to construct artificial tissues to promote tissue regeneration and repair.

7. **Imaging agents**: Nanoparticles can enhance the contrast of medical imaging techniques such as MRI and CT scans.

Nanomedicine enables great accuracy in the treatment of illness with increased targeting and chemical sensitivity. Nanomaterials will undoubtedly have a crucial role to play for future personalized medicine, from prediction to monitoring. Due to it's benefit-to-risk ratio, the use of nanotechnology in health care has become the subject of global discussion. Nanotechnology has severe social and economic effects and little public awareness of it's threats. Advances in the loading and release of drugs are some of the future challenges in this field. Proper evaluation must be carried out before treating patients with full potential, toxicity evaluation, and multistage clinical studies must be performed. Overall, the future of nanotechnology in medicine and healthcare holds a great promise for improving the outcomes of patient and revolutionizing the way we approach disease prevention and treatment.

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Editorial Team

Dr. Timir Karmakar (Assistant Professor)

Bankitdor M. Nongrum (Research Scholar, P22MA008)

Dixita Sonowal (Research Scholar, P23MA001)

Sanchita Pramanik (Research Scholar, P23MA002)

Dibyasman Sarma (Research Scholar, P22MA007)

CONTACT US

Department of Mathematics

National Institute of Technology Meghalaya

Saitsohpen, Sohra (Cherrapunji)-793108

Meghalaya, India

Phone No: 0364-2501294, FAX: 0364-2501113

Editor’s Contact Number: +91 8101058282